General circulation of the atmosphere

Further problems

Problem 1.

In class, we derived a condition on the atmospheric flow known as Hide's theorem.

a) Describe how Hide's theorem constrains the possible atmospheric flows that may exist on Earth. Under what conditions is it valid?

b) Assuming Hide's theorem is applicable, derive a set of limits on the allowable zonal velocity in the atmosphere as a function of latitude. Sketch a plot of these limits (i.e., a plot of zonal wind against latitude) indicating the region of the graph allowable according to Hide's theorem.

In class, we described a state of the atmosphere with no large-scale circulations called radiative-convective equilibrium (RCE). In RCE, the temperature is determined locally by a balance between radiative and convective heating. Consider the vertically averaged temperature of the atmosphere in RCE,

$$\hat{T} = \frac{1}{\ln\left(\frac{p_s}{p_t}\right)} \int_{p_t}^{p_s} T \, d \ln p,$$

where T is the temperature, p is the pressure with values p_s and p_t at the surface and tropopause, respectively.

Suppose that for a given atmosphere the RCE state has a vertically averaged temperature given by

$$\hat{T} = T_0 + \Delta T_0 \exp\left\{\frac{-(\phi - \phi_0)^2}{2\Delta\phi^2}\right\}, \tag{1}$$

where ϕ is the latitude, $\phi_0 = \frac{\pi}{6}$, $\Delta\phi = \frac{\pi}{12}$, and T_0 and ΔT_0 are constants. This distribution is plotted in Fig. 1.

c) For what values of ΔT_0 does this solution violate Hide's theorem? Explain how you arrived at this answer.

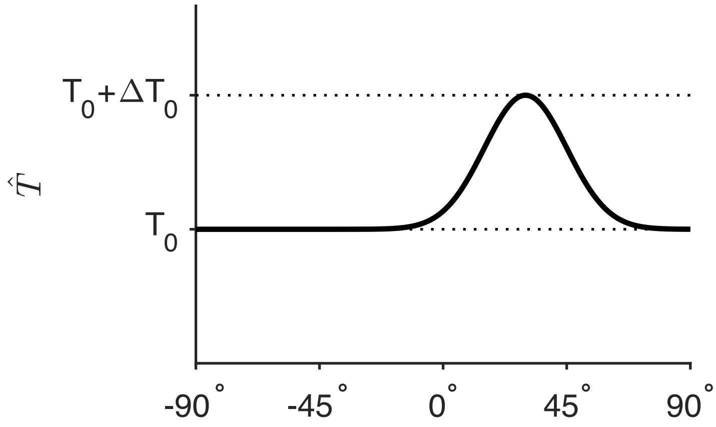

Fig. 1: Vertically-averaged mean temperature in RCE as a function of latitude following (1).

Suppose the RCE solution breaks down, and in its place a Hadley Cell forms. Assume that air rises in the Hadley Cell at $\phi = \phi_0$, flowing southward and northward at the tropopause, conserving angular momentum.

d) Calculate the resulting zonal wind speed at the tropopause as a function of latitude. As before, you may assume the zonal wind at the surface is weak.

Problem 2.

Fig. 2 shows a schematic of the meridional fluxes of angular momentum owing to eddy motions in the southern hemisphere.

a) Sketch the zonal-mean upper tropospheric meridional flow (i.e., $[\overline{v}]$) associated with this distribution of eddy fluxes. Use the angular-momentum budget discussed in class to explain how you arrived at your answer. What assumptions did you need to make? Are these assumptions equally valid in all regions?

In the atmosphere (and in Fig. 2) fluxes of relative angular momentum are weak in the boundary layer.

b) Describe (in words) the balance of terms in the angular-momentum budget within the boundary layer.

Assume that, for zonal flow near the Earth's surface, the deceleration of the wind by friction may be expressed as,

$$F_{\nu\lambda} = -ku,$$

for some constant k.

c) Using this parameterisation, and the balance in (b), derive an equation relating the mean zonal and meridional flow in the boundary layer. For this problem, you may neglect form drag. [Hint: use Eq (5.7) of the notes and the assumption that fluxes of relative angular momentum are weak in the boundary layer.]

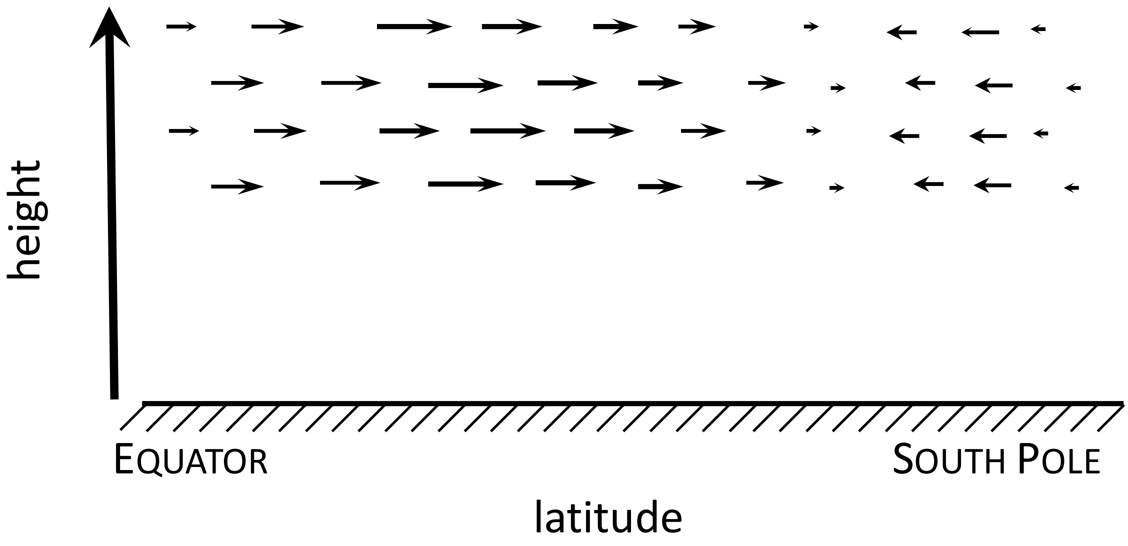

Fig. 2: Schematic of the meridional eddy fluxes of angular momentum $[\overline{v}^*\overline{M}^*] + [\overline{v'M'}]$ as a function of latitude and pressure for the southern hemisphere.

d) Based on part (c) above, sketch the pattern of surface winds associated with the eddy momentum fluxes in the Fig. 2. Explain how you arrived at your sketch.

Problem 3.

Fig. 6.6 of the notes shows the zonal-mean wind at 200 hPa for the Austral winter. Suppose a stationary Rossby wave forms at a latitude of 40°S. Assume this wave is governed by the dispersion relation for barotropic Rossby waves,

$$\omega = k[u] - \frac{k\beta}{k^2 + l^2}$$

where $[u]$ is the zonal-mean zonal wind (which we take as the wind at 200 hPa), β is the gradient in the Coriolis parameter, ω is the frequency of the wave, and k and l are the zonal and meridional wavenumbers, respectively. Assume that $l < 0$.

a) Explain what is meant by the term "stationary" when applied to Rossby waves. Explain why a stationary Rossby wave can propagate energy meridionally.

b) What is meant by the term "critical latitude"? What criterion identifies the critical latitude for a barotropic Rossby wave?

c) Estimate the critical latitude for the wave described above.

d) Describe what happens to the wave's amplitude, and its meridional group and phase velocity as it approaches the critical latitude.

e) Describe the EP fluxes associated with this wave. Where is the associated EP flux divergence positive? Where is it negative?

f) Describe the effect of this wave has on the upper tropospheric zonal wind distribution.

Problem 4.

In the Held-Hou model of the Hadley Cell, air flows poleward from the equator at the tropopause conserving its angular momentum.

a) Assuming the zonal wind is zero at the equator, calculate the zonal wind at a latitude of 20°S according to this model.

According to reanalysis, the annual- and zonal-mean zonal wind at 20°S and 200 hPa is roughly 20 m/s.

b) What process accounts for the bulk of the difference between this number and your answer to part (a)? Explain why the action of this process reduces/increases the zonal wind from its angular-momentum conserving value.

In class, we defined a "local" value of the Rossby number as

$$\mathrm{Ro}^* = -\frac{[\zeta]}{f},$$

where $[\zeta] \approx -\frac{\partial u}{\partial y}$ is the zonal- and time-mean relative vorticity under the tangent plane approximation.

c) Assuming the zonal wind is close to zero at the equator, use finite differences to construct a crude estimate of the relative vorticity at 10°S. Based on this, estimate the Rossby number in the Hadley Cell's upper branch.

d) In class, we derived an equation for the meridional flow in the atmosphere which may be written,

$$f[\overline{v}](1 - Ro^*) = -S$$

where S is proportional to the eddy-momentum flux convergence. Describe the behaviour of the Hadley Cell in the limit $\mathrm{Ro}^* \ll 1$ and in the limit in which $\mathrm{Ro}^* \approx 1$. In particular, explain the influence of midlatitude eddies on the strength of the cell in each case. Under which limit is the Held & Hou (1980) model valid? Based on your answer in (c), is Earth's annual-mean Hadley Cell in either limit?

Problem 5.

Consider a planet similar to Earth in all respects except that it has a very high obliquity (axial tilt). When the obliquity is high, the incoming annual-mean incoming solar radiation peaks at the poles rather than at the equator. Suppose this planet has a radiative-convective equilibrium temperature profile that (in the annual mean) is given by,

$$\hat{T} = T_0 - \Delta T_0 \cos^2 \phi.$$

Suppose, like Earth, this planet has a single region of strong meridional temperature gradients and strong baroclinic eddies at midlatitudes, Describe the main features of the general circulation of this planet. In particular,

a) Sketch the pattern of upper tropospheric winds as a function of latitude

b) Sketch the pattern of surface winds as a function of latitude

c) Sketch the overturning streamfunction as a function of latitude and pressure.

Explain how you arrived at your answers.