General circulation of the atmosphere

Problem set 4

Problem 1.

Consider an inviscid, homogenous, single-layer flow, in which the density is constant $\rho = \rho_0$, and the horizontal velocities are independent of height $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$. Under these conditions, we may write the primitive equations for the zonal and meridional velocity as,

$$\frac{Du}{Dt} = fv + \frac{uv}{R_e}\tan\phi - \frac{1}{\rho_0 R_e \cos\phi}\frac{\partial p}{\partial \lambda},$$

$$\frac{Dv}{Dt} = -fu - \frac{u^2}{R_e}\tan\phi - \frac{1}{\rho_0 R_e}\frac{\partial p}{\partial \phi}.$$

Show that the vorticity equation for such a fluid may be written,

$$\frac{D(f+\zeta)}{Dt} = -(f+\zeta)\nabla\cdot\boldsymbol{u}_h,$$

where $\boldsymbol{u}_h$ is the horizontal velocity, and the vorticity is defined,

$$\zeta = \frac{1}{R_e\cos\phi}\left(\frac{\partial v}{\partial \lambda} - \frac{\partial u\cos\phi}{\partial \phi}\right).$$

Problem 2.

Consider the quasi-geostrophic system on an β-plane. Remember, in this system we define a base state with a potential temperature $\theta_0(p)$ and thermal stratification

$$\frac{\partial \theta_0}{\partial p} = \sigma.$$

Consider a steady propagating wave that creates a geopotential anomaly of the form

$$\Phi^*(x,y,p,t) = A\sin(kx + ly + mp - \omega t),$$

For wavenumbers k, l, and m and angular frequency ω. Here, the asterisk refers a perturbation about a zonally-symmetric steady state.

a) Calculate the perturbation geostrophic zonal and meridional velocities (u_g^*, v_g^*) associated with this wave as a function of (x, y, p, t).
b) The geopotential is related to the potential temperature by the equation for hydrostatic balance:

$$\frac{\partial \Phi}{\partial p} = -\alpha$$

(Note that in this equation the base state relationship $\frac{\partial \Phi_0}{\partial p} = -\alpha_0$ has already been subtracted).

Show that we may write hydrostatic balance for the perturbation geopotential as,

$$\frac{1}{S}\frac{\partial \Phi^*}{\partial p} = -\frac{\theta^*}{\sigma},$$

where $S = \frac{\sigma R \pi}{p}$. What assumption do you need to make about the zonal-mean state?

c) Based on the above equation, calculate the potential temperature anomaly associated with the wave as a function of (x, y, p, t).

d) Using your answers above, calculate the Eliassen-Palm flux for the wave, defined,

$$\boldsymbol{F} = \left(-[u_g^* v_g^*], \frac{f_0[v_g^* \theta^*]}{\sigma}\right)$$

Suppose the wave is a Baroclinic Rossby wave. The dispersion relation for such waves may be written,

$$\omega = [u]k - \frac{\beta k}{\kappa^2}$$

where $\kappa^2 = k^2 + l^2 + \frac{m^2 f_0^2}{S}$, where k, l, and m are the zonal, meridional, and vertical wavenumbers respectively.

e) Calculate the group velocity of the waves. Remember, the group velocity is the velocity at which wave energy (specifically wave activity) is propagated.

f) Compare your answers to d) and e) above. Are there similarities? Think about what this implies for the ability of waves to affect the mean flow.

Problem 3.

Consider the distribution of zonal-mean zonal wind at 200 hPa in the figure below. Suppose a Rossby wave forms at a latitude of 60°S with a zonal phase speed of 10 m s^{-1}, and a positive meridional phase velocity. In the following, assume that the Rossby wave is barotropic, and that the relevant mean zonal background flow is the wind at 200 hPa.

a) Assuming the zonal wavelength is "long", so that $k \to 0$, estimate the meridional wavelength of the Rossby wave.

b) Use linear theory (in combination with the WKB approximation) to estimate the latitude of the *critical line* at which point the wave can no longer propagate.

c) Based on your answers to (a) and (b) above, comment on the validity of the WKB approximation for this problem.

d) In reality, Rossby waves are not small perturbations on the zonal-mean flow. Describe what happens to a fully non-linear wave-like disturbance as it propagates meridionally from 60°S and approaches its critical line.