General circulation of the atmosphere

Problem set 3

Problem 1. (Problem 4 from last week)

The sensible heat flux across a given latitude circle is given by

$$F_H = c_p[vT],$$

where c_p is the isobaric specific heat capacity, v is the meridional velocity and T is the temperature.

Suppose the flow at a given latitude is geostrophic.

a) Show that the heat flux by the zonal-mean flow $[v][T] = 0$.

Suppose at a given time the meridional velocity and temperature at a particular latitude and height may be represented by,

$$v = v_0 \cos(\lambda)$$
$$T = T_0 + \Delta T \cos(\lambda + \eta)$$

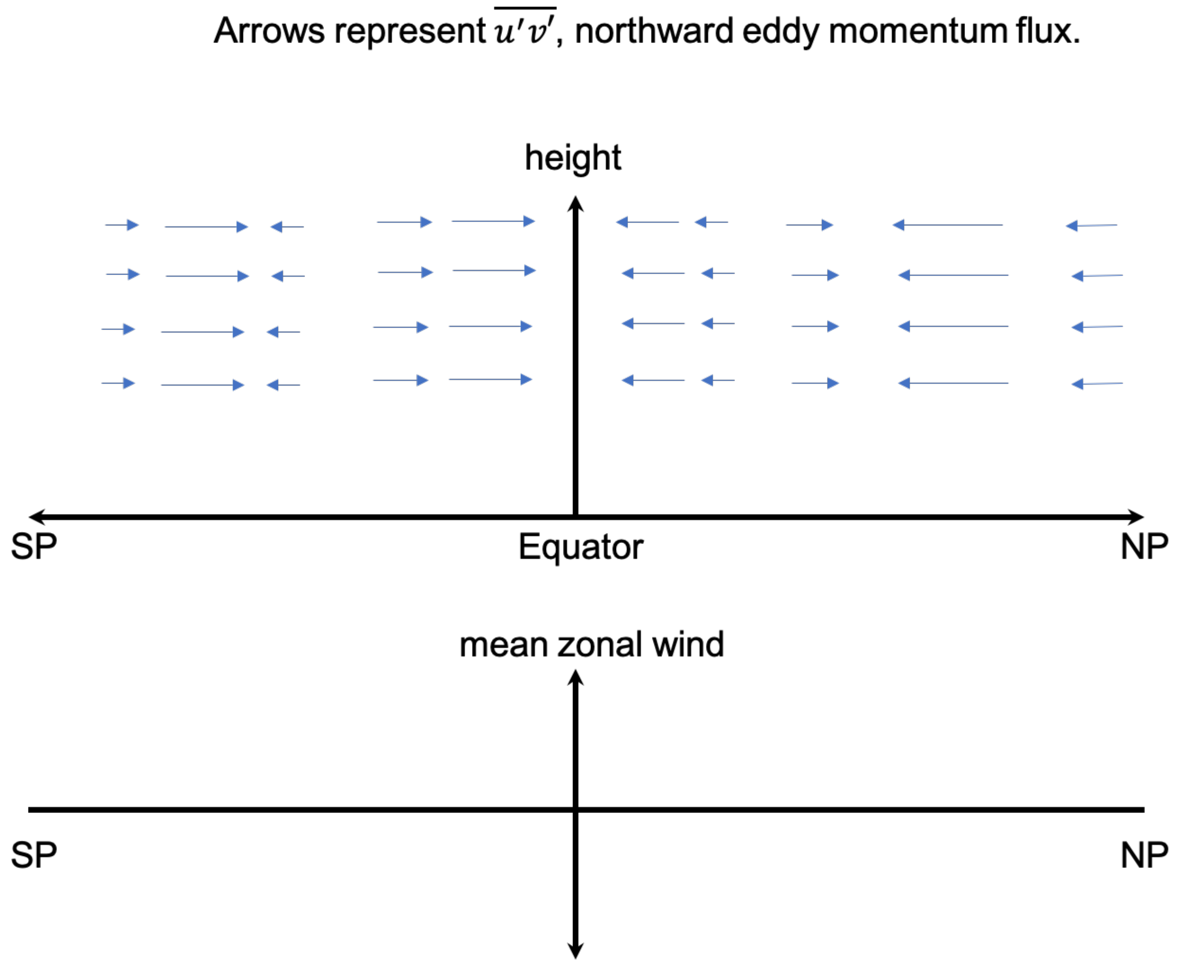
b) Determine the sensible heat flux owing to mean motion $[v][T]$.

c) Calculate the sensible heat flux due to eddy motion $[v^* T^*]$. How does this depend on the phase angle η.

d) If the flow is assumed to be in geostrophic and hydrostatic balance, the temperature is assumed to be uniform in the vertical, and the pressure is assumed to be 1000 hPa and constant at the surface ($z = 0$), determine the value of η and therefore the magnitude of the heat flux at the pressure level 500 hPa and at a latitude of 30 degrees.

Problem 2.

The schematic below shows the eddy-momentum fluxes in a hypothetical atmosphere as a function of latitude (x-axis) and height (y-axis). Arrows give the value of the zonal- and time-mean eddy momentum flux at a given latitude and height up to the tropopause (arrows to the right give northward fluxes, arrows to the left give southward fluxes). Assuming quasi-geostrophic scaling is valid (except very close to the equator), sketch the corresponding overturning streamfunction (on the same axis) and the corresponding zonal-mean surface winds (on the axis below).



Problem 3.

The inviscid momentum equation written in an inertial reference frame is given by,

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{1}{\rho}\nabla p - \nabla\Phi_g,$$

following the notation of the class notes. Consider the circulation of a closed material contour $l(t)$,

$$\Gamma(t) = \oint_{l(t)} \boldsymbol{u}\cdot d\boldsymbol{r}.$$

Suppose we may parameterise the path l by a scalar s, so that at a given time t we have,

$$l(s;t) = \{\boldsymbol{r}(s;t) = (x(s;t), y(s;t), z(s;t)); s \in [0,1]\}.$$

Since l is a material contour, this implies that

$$\left(\frac{\partial \boldsymbol{r}}{\partial t}\right)_s = \frac{D\boldsymbol{r}}{Dt}.$$

We may then write,

$$\Gamma(t) = \int_0^1 \boldsymbol{u}\cdot\left(\frac{\partial \boldsymbol{r}}{\partial s}\right)_t ds.$$

a) Use the momentum equation to show that

$$\frac{d\Gamma(t)}{dt} = 0$$

provided that the density is constant. This is Kelvin's circulation theorem.

b) Further, show that Kelvin's circulation theorem is still valid provided the pressure is a function of density only. This is what is known as a barotropic fluid.

c) Show that in a barotropic and geostrophic fluid, there can be no horizontal (at constant pressure) temperature gradients.