General circulation of the atmosphere

Problem set 2

Problem 1.

Consider the evolution equation for a conserved scalar quantity γ,

$$\frac{D\gamma}{Dt} = 0. \tag{1}$$

a) Use the continuity equation to show that the (1) may be expressed in flux form as,

$$\frac{\partial \rho\gamma}{\partial t} + \nabla \cdot (\rho\gamma\boldsymbol{u}) = 0, \tag{2}$$

where ρ is the density and $\boldsymbol{u}$ is the vector velocity.

b) Take the time mean of (2), thereby deriving an equation for the zonal and time mean value of γ. Show that this equation may be written,

$$\frac{D\overline{\gamma}}{Dt} = -\frac{1}{\overline{\rho}}\nabla \cdot \left(\overline{\rho}\,\overline{\gamma'\boldsymbol{u}'}\right), \tag{3}$$

where we have neglected time and zonal variations in density.

c) Explain the differences between (1) and (3). That is, what is the meaning of the extra term on the right-hand side in the equation for the time-mean value $\overline{\gamma}$ compared to the equivalent equation for γ.

Problem 2.

Consider the covariance between two arbitrary variables AB.

a) Show that we may write the zonal and time mean of this covariance as,

$$[\overline{AB}] = [\overline{A}][\overline{B}] + \overline{[A]'[B]'} + [\overline{A^*B^*}].$$

b) Explain how this differs from the decomposition we constructed in class. Which do you think is more useful and why?

Problem 3.

In class this week, we derived an equation for the zonal-mean meridional flow in the upper troposphere which may be written,

$$f[\overline{v}] = \frac{1}{R_e \cos^2\phi}\frac{\partial}{\partial\phi}([\overline{uv}]\cos^2\phi) + \frac{\partial}{\partial p}([\overline{u\omega}]). \qquad (4)$$

This equation relates the upper tropospheric meridional flow to the flux divergence of angular momentum (by both eddies and the mean flow).

On the other hand, in the Held & Hou model of the Hadley Cell, the strength of the cell (which is related to the strength of the meridional flow in the upper troposphere) was determined using the thermodynamic balance.

Discuss how this difference in our perspective arises. How is (4) satisfied in the (axisymmetric) Held & Hou model? What happens to (4) when eddies are important?

Problem 4.

The sensible heat flux across a given latitude circle is given by

$$F_H = c_p[vT],$$

where c_p is the isobaric specific heat capacity, v is the meridional velocity and T is the temperature.

Suppose the flow at a given latitude is geostrophic.

a) Show that the heat flux by the zonal-mean flow $[v][T] = 0$.

Suppose at a given time the meridional velocity and temperature at a particular latitude and height may be represented by,

$$v = v_0\cos(\lambda)$$
$$T = T_0 + \Delta T\cos(\lambda + \eta)$$

b) Determine the sensible heat flux owing to mean motion $[v][T]$.

c) Calculate the sensible heat flux due to eddy motion $[v^*T^*]$. How does this depend on the phase angle η.

d) If the flow is assumed to be in geostrophic and hydrostatic balance, the temperature is assumed to be uniform in the vertical, and the pressure is assumed to be 1000 hPa and constant at the surface ($z = 0$), determine the value of η and therefore the magnitude of the heat flux at the pressure level 500 hPa and at a latitude of 30 degrees.