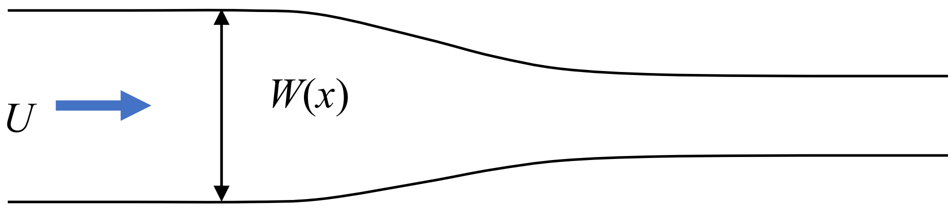
General circulation of the atmosphere

Problem set 1

Problem 1.



Consider steady flow of water through a pipe with constant height H and variable width,

$$W(x) = W_0\left(1 - \frac{1}{2}\tanh x\right),$$

where $W_0 = 1$ m. At the upstream end of the pipe ($x \to -\infty$), the average velocity of the flow within the pipe is equal to $U = 1$ m s^{-1}.

a) Calculate the velocity at the downstream end of the pipe ($x \to \infty$).

b) Calculate the rate of change of velocity for a parcel or fluid at the point $x = 0$.

c) Explain why the parcel experiences acceleration even though the flow is steady.

d) Assuming the flow is frictionless, estimate the horizontal pressure gradient at the location $x = 0$. Assume the density is 1000 kg m^{-3}

Problem 2.

Show that for the spherical coordinate system defined in lectures,

$$\frac{D\mathbf{u}}{Dt} = \left(\frac{Du}{Dt} - \frac{uv\tan\phi}{r} + \frac{uw}{r}\right)\hat{\boldsymbol{\lambda}} + \left(\frac{Dv}{Dt} + \frac{u^2\tan\phi}{r} - \frac{vw}{r}\right)\hat{\boldsymbol{\phi}} + \left(\frac{Dw}{Dt} - \frac{u^2+v^2}{r}\right)\hat{\boldsymbol{r}}$$

Hint: calculate the rate of change of the basis vectors.

Problem 3.

Define the potential temperature by

$$\theta = T\left(\frac{p}{p_0}\right)^{R/c_p}$$

a) Show that this quantity is conserved for adiabatic displacements.

Consider an atmosphere at rest with a potential temperature distribution equal to $\theta_0(z)$. Suppose an air parcel is displaced vertically from its resting position a small distance δz.

b) Assuming adiabatic air motion, estimate the buoyancy of the displaced air parcel. Here, the buoyancy is defined,

$$b = -\frac{g(\rho - \rho_0)}{\rho}.$$

A simplified evolution equation for the vertical velocity w relates vertical accelerations to the buoyancy,

$$\frac{Dw}{Dt} = b.$$

c) Use this equation and your answer to (b) to calculate the evolution of the vertical velocity as a function of time for small displacements. What are the characteristics of the resulting motion? How does this depend on the vertical gradient of the potential temperature?

Problem 4.

The horizontal momentum equations for frictionless flow on an *f*-plane may be written,

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x},$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial x},$$

where f is the (constant) Coriolis parameter). Consider a state that has no pressure gradients. Within this background state, a parcel of air has an initial velocity (u_0,0). Solve for the parcel's position as a function of time. What shape does the parcel trace out in space? Is the motion oscillatory? If so, what is the period of the oscillation? How does this depend on the latitude at which the parcel is moving?