

# Vertical structure of warming consistent with an upward shift in the middle and upper troposphere

## Supplementary Information

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### Application of the vertical shift transformation (VST) to the RCP8.5 simulations

We apply the full form of the VST to the RCP8.5 simulations because the temperature changes are sufficiently large that the linearized version of the VST gives noticeably different results. (The tropical temperature changes in the historical simulations correspond to  $\beta - 1 \simeq 0.03$ , whereas for the RCP8.5 simulations they correspond to  $\beta - 1 \simeq 0.2$ .) The transformed temperature  $T'$  is given in terms of the original temperature  $T$  by

$$T'(p) = T(\beta p) - \Delta\theta \Pi(\beta p), \quad (1)$$

where  $\Pi$  is the Exner function and  $\beta$  is the transformation parameter (Singh and O’Gorman 2012).

The potential temperature offset  $\Delta\theta$  arises from the effects of water vapor and is given in terms of  $\beta$  as

$$\Delta\theta = \left(\frac{\beta - 1}{\beta}\right) \left(\frac{R_v}{L}\right) T\theta, \quad (2)$$

where  $R_v$  is the gas constant for water vapor and  $L$  is the latent heat of vaporization or sublimation of water. The theory assumes that fractional variations in  $\Delta\theta$  are small, and Singh and O’Gorman 2012 evaluated it at a fixed pressure level of 600hPa. We make a slightly different approximation by evaluating  $\Delta\theta$  locally at  $\beta p$  which results in a simpler expression for the transformed temperature

as

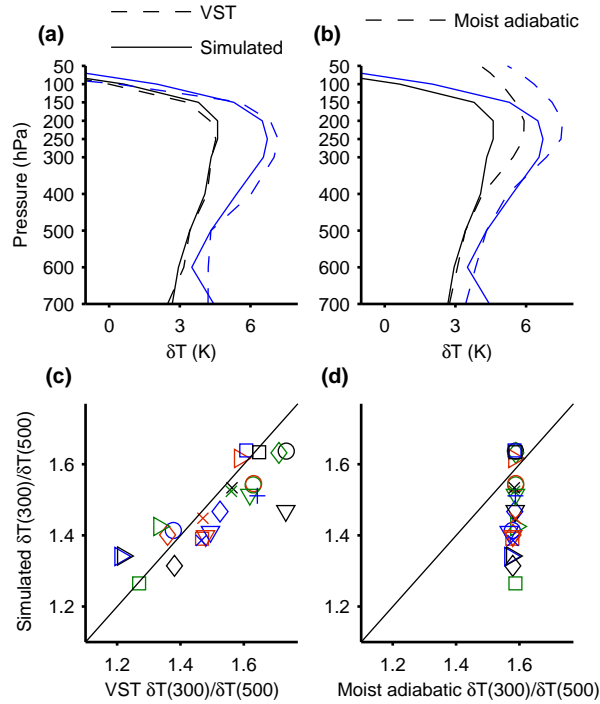
$$T'(p) = T(\beta p) - \frac{\beta - 1}{\beta} \frac{R_v}{L} T(\beta p)^2. \quad (3)$$

(Our results are very similar if we follow Singh and O’Gorman 2012 by evaluating  $\Delta\theta$  at a fixed pressure level.)

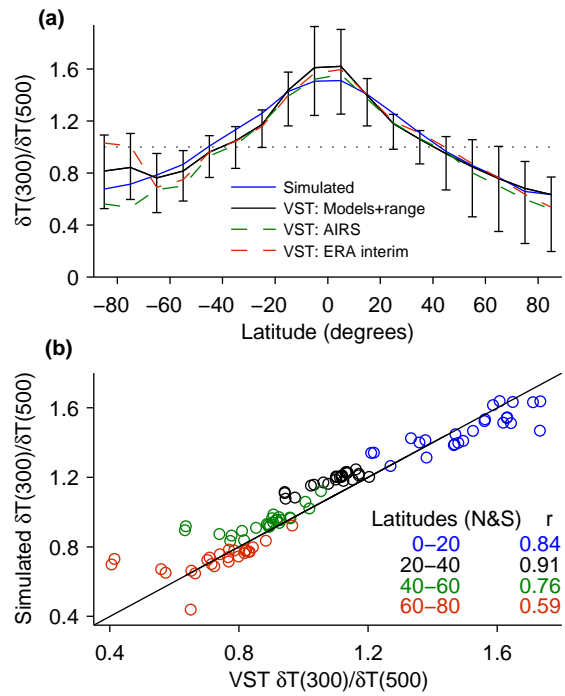
If  $\beta$  is chosen so that the simulated temperature change matches the transformed temperature change  $\delta T_{\text{VST}}(p) = T'(p) - T(p)$  at a given level, then expression (3) gives an estimate of the temperature change for all other levels at which the transformation is valid. In applying the VST to the RCP8.5 simulations, we determine  $\beta$  at each latitude by matching  $\delta T_{\text{VST}}(500\text{hPa})$  to the simulated temperature change at 500hPa, with the temperature at  $\beta p$  in expression (3) evaluated using cubic-spline interpolation. The resulting value of  $\beta$  at each latitude is then used to determine the transformed temperature changes at other levels. The VST is applied to mean temperatures from the control climate (the historical simulations averaged from 1960-2005), and the resulting transformed temperatures are compared to mean temperatures from the warm climate (the RCP8.5 simulations averaged from 2081-2100). The exact time periods used are different for some models and are given in Table S1. Results for RCP8.5 are shown in Figs. S1 and S2.

	Model	Historical	RCP8.5
○	BCC-CSM1-1	1960–2005	2080–2099
○	BNU-ESM	1960–2005	2081–2100
○	CCSM4	1960–2005	2081–2100
○	CESM1-BGC	1960–2005	2081–2100
◇	CESM1-CAM5	1960–2005	2081–2100
◇	CNRM-CM5	1960–2005	2081–2100
◇	FIO-ESM	1960–2005	2081–2100
◇	GFDL-CM3	1960–2005	2081–2100
▷	GFDL-ESM2G	1960–2005	2081–2100
▷	GFDL-ESM2M	1960–2005	2081–2100
▷	GISS-E2-R	1960–2005	2081–2100
▷	HadGEM2-CC	1960–2004	2080–2099
□	HadGEM2-ES	1960–2004	2081–2099
□	HadGEM2-AO	1960–2005	2080–2099
□	INMCM4	1960–2005	2081–2100
□	IPSL-CM5A-LR	1960–2005	2081–2100
▽	IPSL-CM5A-MR	1960–2005	2081–2100
▽	IPSL-CM5B-LR	1960–2005	2081–2100
▽	MIROC5	1960–2005	2080–2099
▽	MIROC-ESM-CHEM	1960–2005	2081–2100
×	MIROC-ESM	1960–2005	2081–2100
×	MPI-ESM-LR	1960–2005	2081–2100
×	MPI-ESM-MR	1960–2005	2081–2100
×	MRI-CGCM3	1960–2005	2081–2100
+	NorESM1-M	1960–2005	2081–2100

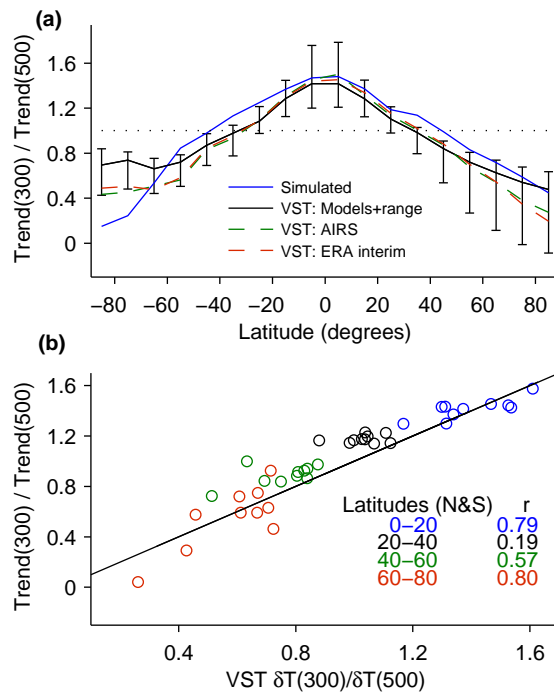
**Table S1.** CMIP5 model identifiers and the corresponding time periods in the historical and RCP8.5 simulations. Markers on the left correspond to the plotting symbols in Figs. 1c and d.



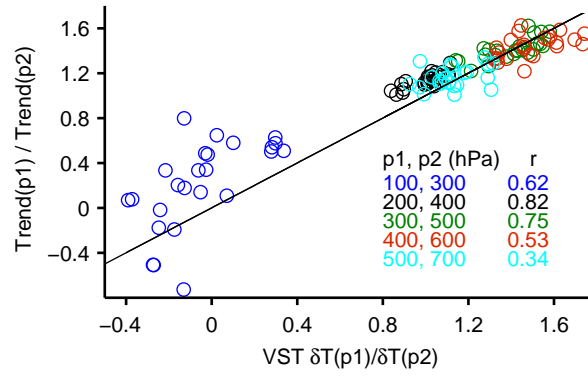
**Figure S1.** As in Fig. 1 but for temperature changes under RCP8.5 rather than for historical trends (see supplementary text for details). The moist-adiabatic warming profiles are calculated in this case by integrating a saturated moist adiabat from 500hPa in pressure (to both higher and lower pressure levels) in the control and warm climates and then taking the difference in temperature at each level. Unlike in Fig. 1, rescaling (for presentation) of the VST and moist-adiabatic profiles is not needed. The models shown in (a,b) are the same as the models shown in Fig. 1 to allow for comparison (INMCM4 in black and MIROC5 in blue).



**Figure S2.** As in Fig. 2 but for temperature changes under RCP8.5 rather than for historical trends (see supplementary text for details). In calculating the VST for AIRS and ERA-interim in panel (a), the transformation parameter  $\beta$  is calculated using expression (3) evaluated at 500hPa at each latitude with a nominal temperature increase of 4.9K. This temperature increase is the multimodel- and global-mean warming at 500hPa for the climate-model simulations shown in this figure.



**Figure S3.** As in Fig. 2 but using historical simulations with only greenhouse-gas forcing. The greenhouse-gas-only simulations were only available for the BCC-CSM1-1, CCSM4, CNRM-CM5, GFDL-CM3, GFDL-ESM2M, HadGEM2-ES, MIROC-ESM-CHEM, MIROC-ESM, MRI-CGCM3, and NorESM1-M models.



**Figure S4.** Ratios of tropical temperature trends as in Fig. 1c, but for a range of pairs of pressure levels ( $p1, p2$ ) rather than just 300hPa and 500hPa. The correlation coefficient ( $r$ ) across models for each pair of pressure levels is given in the legend.