Lecture 3 : Radiative-convective equilibrium

Why is there an atmospheric circulation?

This lecture: Begin with assumption of no motion, see the implications

Radiative equilibrium

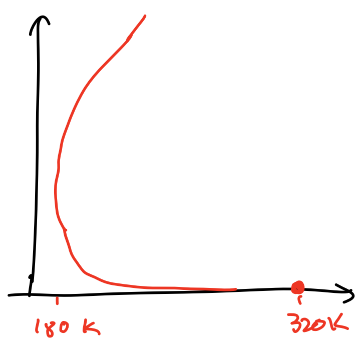
- Suppose atmosphere not allowed to move.

What would be the characteristics of the climate?

- Only energy transport through radiation
- Assume steady state, must have

$$\frac{\partial}{\partial z}(F_{LW} + F_{SW}) = 0$$

- Calculation performed by Manabe & Strickler (1964) for tropics



⇒ large ΔT between surf & atmosphere

⇒ large lapse rate in lower troposphere

What would happen if started from this state?

This state is convectively unstable

⇒ Expect convection to begin to stabilise atmosphere

Radiative - convective equilibrium

Balance between convection & radiation:

$$Q_{rad} + Q_{conv} = 0$$

Convection is a "fast" process relative to radiation

⇒ convective instability rapidly removed

atmosphere organises itself into state neutral to convection

What is such a state?

→ As air parcels rise, they do not feel positive or negative buoyancy

Dry atmosphere

When air parcel rises what happens to temperature?

In dry atmosphere (no phase change), air parcels conserve θ

∴ Neutral state is one in which

$$\frac{\partial \theta}{\partial z} = 0$$

also $\frac{\partial s}{\partial z} = 0$ $\quad s = c_p \ln(\theta) =$ dry entropy

Moist atmosphere

Why is θ not conserved for airparcels in moist atmosphere

Assessing convective stability is more complicated

→ existence of conditional instability What is this

→ buoyancy depends where parcel is lifted from

Surf driven convection ⇒ Expect virtual temperature of environment to be close to that of parcel raised from near surface.

In sub-cloud layer:

→ no condensation

→ same as dry convection $\frac{\partial \theta}{\partial z} = 0$

In cloud:

→ parcel at saturation

→ roughly conserves saturation equivalent potential temperature

$$\theta_e^* \simeq \theta \exp\left(\frac{r^* L_v}{c_p T} \right)$$

→ θ_e is related to entropy

$$s = c_p \ln(\theta_e)$$

⇒ Note: this depends on a particular meteorological choice to set the reference entropy of liquid water to zero at some reference state.

Entropy

What is entropy? what affects entropy?

- Conserved for reversible transformations
- Not conserved for
  - precipitation fallout
  - mixing
  - freezing $\neq 0°C$, melting $\neq 0°C$
  - frictional dissipation
- Not conserved if external heating is applied
- Critical to applying second law of thermodynamics to climate system

  see: Singh & O'Neill (2022), Rev. Mod. Phys.

Thermal structure of RCE

1) Assume cloud buoyancy is small:

$$T_v^{cloud} \simeq T_v^{environment} = T_v^{average}$$

2) Assume clouds are rising undilute bubbles

$\Rightarrow$ Expect saturation moist entropy to be constant above subcloud layer in RCE.

i.e., lapse rate follows moist adiabat

{ Aside: Actual lapse-rate deviates from moist adiabatic see Singh & O'Gorman (2013) }

Tropical thermodynamic structure

Observations indicate that $\frac{\partial \theta_e^*}{\partial z} \simeq 0$ in the tropics.

This is a result of two effects:

1) Convective quasi-equilibrium

Convecting regions rapidly adjust to neutrality to moist convection

2) Weak temperature gradient approximation

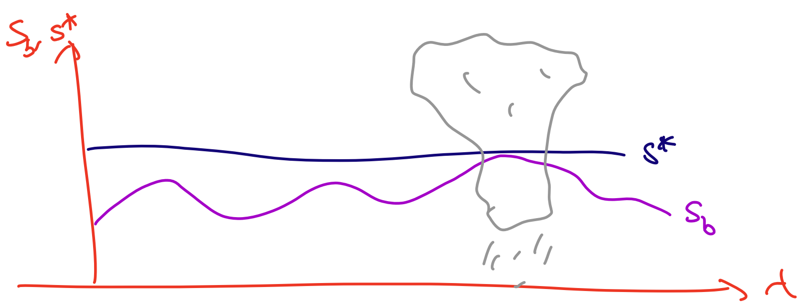
In deep tropics f is "small", so strong temperature gradients cannot be maintained

$$f \frac{\partial u}{\partial p} = \frac{R}{p} \frac{\partial T}{\partial y}$$

$$f \to 0 \qquad \frac{\partial T}{\partial y} \to 0$$

Expect $s^* \approx s_b$ in convecting regions of the tropics.

This value of s^* is propagated to other regions via gravity waves.



Implications: - In regions of strong convection θ_e^* in troposphere closely tied to θ_e in boundary layer

- In regions where θ_e is low, convection is surpressed

$\Rightarrow$ Ability of atmosphere to support convection depends strongly on subcloud entropy (θ_e).

A solution to the large-scale equations?

Have seen radiative equilibrium is unstable

$\Rightarrow$ atmosphere must move

But what about radiative-convective equilibrium as a global solution.

1) Hydrostatic balance
2) Column-by-column RCE
3) No overturning circulation ($v = w = 0$)
4) No longitudinal variations
5) Zonal winds in thermal wind balance
6) ~~zero~~ surface winds
7) inviscid flow above the surface

How does this RCE sol$^{\underline{n}}$ work?

Equations: Thermodynamic : satisfied in each column by RCE assumption (2)

Hydrostatic : satisfied by assumption (1)

u-momentum:

$$\frac{\partial u}{\partial t} + \cancel{\frac{u}{R_e\cos\phi}\frac{\partial u}{\partial \lambda}} + \cancel{\frac{v}{R_e}\frac{\partial u}{\partial \phi}} + \cancel{\omega\frac{\partial u}{\partial p}} = \overset{(3)}{\cancel{2\Omega\sin\phi\, v}} + \overset{(3)}{\cancel{\frac{uv\tan\phi}{R_e}}} - \overset{(4)}{\cancel{\frac{1}{R_e\cos\phi}\frac{\partial \Phi}{\partial \lambda}}} + \overset{(6,7)}{\cancel{F_x}}$$

v-momentum:

$$\cancel{\frac{\partial v}{\partial t}} + \cancel{\frac{u}{R_e\cos\phi}\frac{\partial v}{\partial \lambda}} + \cancel{\frac{v}{R_e}\frac{\partial v}{\partial \phi}} + \overset{(3)}{\cancel{\omega\frac{\partial v}{\partial p}}} = -2\Omega\sin\phi\, u - \frac{u^2\tan\phi}{R_e} - \frac{1}{R_e}\frac{\partial \Phi}{\partial \phi} + \overset{(6,7)}{\cancel{F_y}}$$

$$\frac{1}{R_e\cos\phi}\left(\overset{(4)}{\cancel{\frac{\partial u}{\partial \lambda}}} + \cancel{\frac{\partial v\cos\phi}{\partial \phi}}\right) + \overset{(3)}{\cancel{\frac{\partial \omega}{\partial p}}} = 0 \qquad \checkmark$$

$$-2\Omega\sin\phi\, u - \frac{u^2\tan\phi}{R_e} - \frac{1}{R_e}\frac{\partial \Phi}{\partial \phi} = 0$$

Quadratic to express u in terms of $\frac{\partial \Phi}{\partial \phi}$ can be solved.

Full non-linear solution to primitive equations!

Why is it not observed??