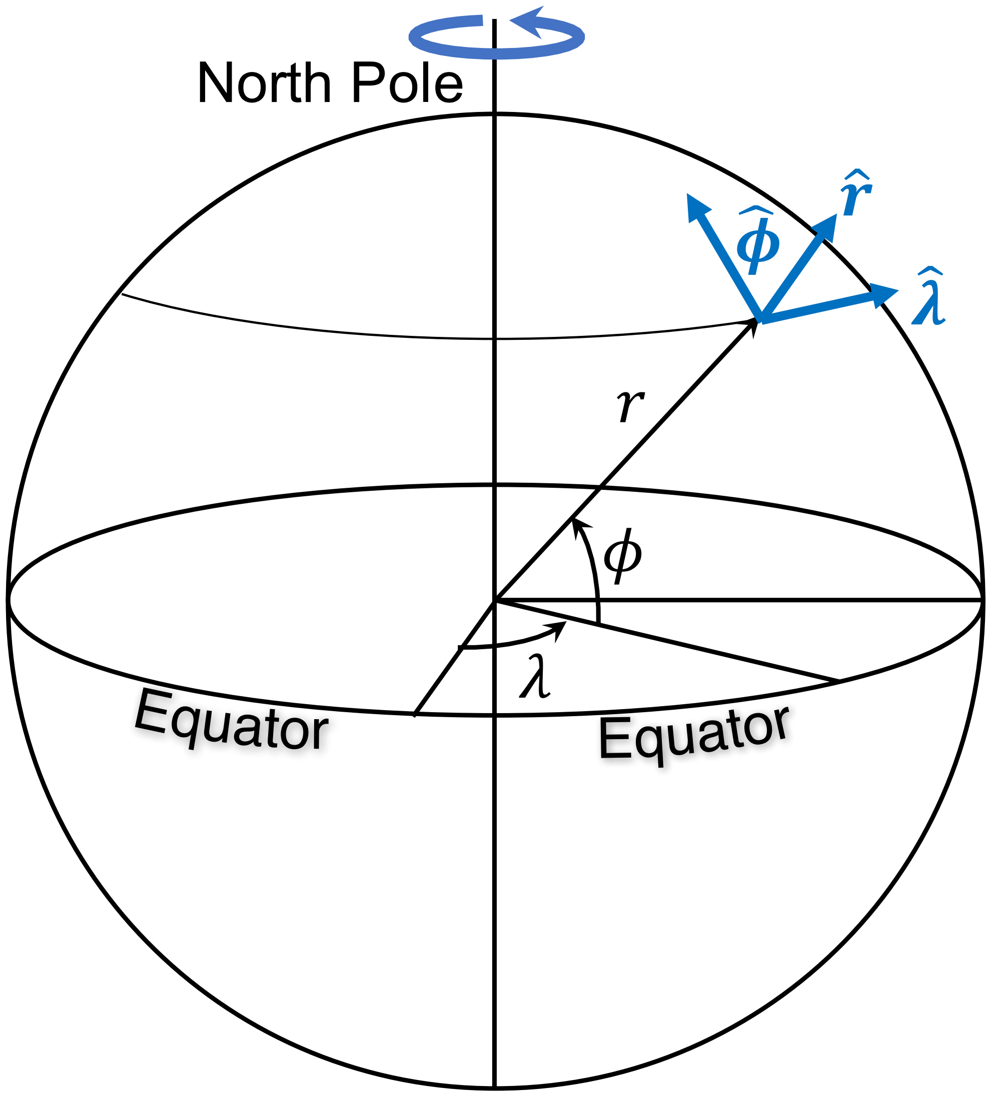
Spherical coordinates

λ: longitude
ϕ: latitude
r: distance from origin



Summary of equations in primitive form

$$\frac{Du}{Dt} = 2\Omega \sin\phi\, v + \frac{uv}{R_e}\tan\phi - \frac{1}{R_e\rho\cos\phi}\frac{\partial p}{\partial\lambda} + F_\lambda \qquad \text{......1}$$

$$\frac{Dv}{Dt} = -2\Omega \sin\phi\, u - \frac{u^2}{R_e}\tan\phi - \frac{1}{R_e\rho}\frac{\partial p}{\partial\phi} + F_\phi \qquad \text{......2}$$

$$\frac{\partial p}{\partial z} = -\rho g \qquad \text{......3}$$

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot\rho\boldsymbol{u} \qquad \text{......4}$$

$$q = c_p\frac{DT}{Dt} - \alpha\frac{Dp}{Dt} \qquad \text{......5}$$

$$p\alpha = R_d T \qquad \text{......6}$$

Where:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{R_e\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{R_e}\frac{\partial}{\partial\phi} + w\frac{\partial}{\partial z}$$

$$\nabla\cdot\boldsymbol{u} = \frac{1}{R_e\cos\phi}\left\{\frac{\partial u}{\partial\lambda} + \frac{\partial}{\partial\phi}(v\cos\phi)\right\} + \frac{\partial w}{\partial z}$$

Summary of tangent plane equations

$$x \sim R_e \cos\phi_0 \, \lambda$$
$$y \sim R_e(\phi - \phi_0)$$

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} + F_x \qquad \text{......1}$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} + F_y \qquad \text{......2}$$

$$\frac{\partial p}{\partial z} = -\rho g \qquad \text{......3}$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \boldsymbol{u} \qquad \text{......4}$$

$$q = c_p \frac{DT}{Dt} - \alpha \frac{Dp}{Dt} \qquad \text{......5}$$

$$p\alpha = R_d T \qquad \text{......6}$$

Where

$$f = f_0 = 2\Omega \sin\phi_0 \qquad (f\text{-plane})$$

or

$$f = f_0 + \beta y \qquad (\beta\text{-plane})$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

$$\nabla \cdot \boldsymbol{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

Summary of pressure-coordinate equations

$$\frac{Du}{Dt} = 2\Omega \sin\phi\, v + \frac{uv}{R_e}\tan\phi - \frac{1}{R_e\cos\phi}\frac{\partial\Phi}{\partial\lambda} + F_\lambda \qquad \text{......1}$$

$$\frac{Dv}{Dt} = -2\Omega \sin\phi\, u - \frac{u^2}{R_e}\tan\phi - \frac{1}{R_e}\frac{\partial\Phi}{\partial\phi} + F_\phi \qquad \text{......2}$$

$$\frac{\partial\Phi}{\partial p} = -\alpha \qquad \text{......3}$$

$$\nabla_p \cdot \boldsymbol{u} = 0 \qquad \text{......4}$$

$$q = c_p\frac{DT}{Dt} - \alpha\omega \qquad \text{......5}$$

$$p\alpha = R_d T \qquad \text{......6}$$

Where:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{R_e\cos\phi}\frac{\partial}{\partial\lambda} + \frac{v}{R_e}\frac{\partial}{\partial\phi} + \omega\frac{\partial}{\partial p}$$

$$\nabla_p \cdot \boldsymbol{u} = \frac{1}{R_e\cos\phi}\left\{\frac{\partial u}{\partial\lambda} + \frac{\partial}{\partial\phi}(v\cos\phi)\right\} + \frac{\partial\omega}{\partial p}$$

Can also write these equations in tangent plane form.