General circulation of the atmosphere

Assignment 1

Due date: 5pm Friday 19 April 2024

Instructions

Answer the problems given below and remember to show your working. This assignment is worth 15% of your final mark. The assignment is out of 21 marks.

Problem 1. (10 marks)

(a) Consider the zonal momentum equation in primitive form and written in pressure coordinates:

$$\frac{Du}{Dt} = 2\Omega \sin\phi\, v + \frac{uv}{R_e}\tan\phi - \frac{1}{R_e\cos\phi}\frac{\partial\Phi}{\partial\lambda} + \mathcal{F}_\lambda,$$

where all symbols are as defined in lectures. Show that for axisymmetric, frictionless flow the quantity

$$M = R_e\cos\phi(u + \Omega R_e\cos\phi)$$

is conserved.

(5 marks)

(b) Consider an air parcel at the equator moving upwards. The air is moving away from the axis of rotation of the Earth, and so the moment arm between this axis and the parcel is increasing, and the angular momentum of the parcel about the Earth's rotation axis is therefore also increasing. Despite this, the value of M, which we usually identify as a measure of angular momentum, does not change. Explain why this is the case.

(3 marks)

(c) Hide's theorem states that, under certain conditions, the atmosphere is prevented from having a maximum of angular momentum above the surface. However, Jupiter has a westerly jet in its equatorial upper atmosphere. Why does this flow not violate Hide's theorem?

(2 marks)

Problem 2. (11 marks)

In class, we developed a hypothetical axisymmetric solution to the primitive equations in which each column of the atmosphere is separately in radiative-convective equilibrium and the zonal-wind is in thermal wind balance with the temperature field.

a) Describe what is meant by the term "radiative-convective equilibrium".

(1 mark)

b) Give at least one reason why such a solution is not observed, even in the limit in which the viscosity of the atmosphere approaches zero. Explain your answer.

(2 marks)

Suppose that the log-pressure-weighted mean temperature of the RCE solution is of the form

$$\hat{T} = T_0 + \delta T \exp\left(-\frac{\phi^2}{2(\delta\phi)^2}\right),$$

where

$$\hat{T} = \frac{1}{\ln(p_s/p_t)} \int_{p_t}^{p_s} T \, d\ln p,$$

and p_s and p_t are the pressure at the tropopause and surface, respectively, ϕ is latitude and $T_0, \delta T$ and $\delta\phi$ are constants.

c) Calculate the zonal-wind distribution at the tropopause for this RCE distribution of $\hat{T}$, and plot it for the case $\delta T = 100$ K and $\delta\phi = 30°$.

You may assume that the zonal wind at the surface is zero.

(5 marks)

d) Does this solution violate Hide's theorem? Explain why or why not.

(3 marks)